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GEOMORPHOMETRIC PARAMETERS: A REVIEW AND EVALUATION

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ABSTRACT. All aspects of surface form can be considered to reflect surface roughness. Horizontal variation includes the concepts of texture and grain, while vertical variation is discussed under relief. The relationships between these are embodied in slope and the dispersion of slope magnitude and orientation. The distribution of mass within the elevation range of a topographic surface is described under hypsometry. Parameters for further investigation are selected from these categories after an examination of the relationships among the variables using correlation analysis.

Introduction

Geomorphometry, which has been defined by Chorley *et al.* (1957, p. 138) as the science "which treats the geometry of the landscape," attempts to describe quantitatively the form of the land surface; it is a sub-discipline of geomorphology. Evans (1972, p. 18) distinguished *specific* geomorphometry, which measures the geometry of specific types of landforms (*e.g.* Troeh's 1964, 1965, "landform equations") from *general* geomorphometry, "the measurement and analysis of those characteristics of landforms which are applicable to any continuous rough surface." It has been claimed that the drainage basin represents "the fundamental geomorphic unit" (notably Chorley 1969; also Leopold *et al.* 1964). This view was taken to an extreme by Connelly (1968), who in a discussion of terrain statistics stated that "although it is an oversimplification, it is certainly a valid approximation to attribute all land forms to the fluvial erosion of uplifted rock masses" (p. 78). He stated that this assumption was necessary in order to develop "a unified framework for landscape geometry." Since about one third of the earth's land surface was glaciated during the Pleistocene (*cf.* Flint 1971, p. 19), and as other processes such as fluvial deposition, or aeolian, volcanic, or periglacial action have also influenced large single process is assumed. Furthermore, the specific approach can only be applied once an area has been identified as a drainage basin, an alluvial fan, a drumlin, etc.

This paper reports part of the results of a study whose object was to investigate the use of computer-stored topographic information in the evaluation of geomorphometric parameters. Computers have been widely employed in both geography and the earth sciences, and geomorphology has not been an exception. A recent book edited by Chorley (1972) indicates that spatial aspects of land surface form have received much attention. While computers have been used in geomorphometry, there have been few attempts to store topographic surfaces in computers and then to perform detailed quantitative analyses of land surface form. Exceptions are the works of Hormann (1969, 1971), who approximated land surfaces with sets of contiguous triangles, and of Evans (1972), whose work was based on regular square grids. In this paper, an attempt will be made to review a considerable number of geomorphometric parameters so as to produce a rational classification of these measures. Attention will be focussed upon two points: the amenability of the parameters to measurement based upon computer terrain storage systems, and the probable geomorphic significance of the measures. The classification will thus differ from that of Evans (1972), whose emphasis was upon the relationship of measures to classical statistical parameters rather than to the above points.

For the reason cited above, emphasis will be upon parameters of general geomorphometry, although some attention will be directed toward measures based specifically on landforms of fluvial activity, probably the most important *single* class of processes which has shaped the earth's surface.

could be divided into *geometrical* properties, which involve the relationships among dimensional properties such as elevations, lengths, areas, and volumes, and *topological* properties which relate numbers of objects in the drainage net (*e.g.* the bifurcation ratio). The latter properties will not be considered herein.

All measures of land surface form can be considered to be in some way representative of the "roughness" of the surface. This review will thus begin with a discussion of the general concept of roughness before proceeding to actual geomorphometric parameters. Finally, relationships among the variables will be examined empirically using correlation analysis.

The concept of "roughness"

In a general sense, *roughness* refers to the irregularity of a topographic (or other) surface. Stone and Dugundji (1965) and Hobson (1967) observed that roughness cannot be completely defined by any single measure, but must be represented by a "roughness vector" or set of parameters. One area may be rougher than another because it has a shorter characteristic wavelength, a higher amplitude, an irregularity of ridge spacing, or sharper ridges. Stone and Dugundji, in a study of microrelief profiles, used 5 measures, while Hobson computed 9 others based on three different "roughness concepts".

It is convenient to discuss terrain roughness by analogy with combinations of periodic functions or *spectra* of the terrain. Evans (1972, p. 33—36) reviewed some of the attempts to analyze topography using spectral analysis explicitly. He observed (p. 36) that in practice this has not been very successful because valleys often curve and they converge downstream, while valley spacing within an area is seldom regular. The general ideas of wavelength and amplitude are useful, however, and geomorphometric measures will be discussed in this context. The significant wavelengths of topography are termed *grain* or *texture,* while amplitudes associated with these wavelengths correspond to the concept of *relief*. The relationship between the horizontal and vertical dimensions of the topography is embodied in [illegible] surface is contained in the concept of *hypsometry*.

Texture and Grain

Texture and grain are terms which have been used to indicate in some way the scale of horizontal variations in the topography. These terms have been used in different contexts, and this difference is preserved if *texture* is used to refer to the *shortest* significant wavelength in the topography and *grain* used for the *longest* significant wavelength. Texture is related to the smallest landform elements one wishes to detect, and grain to the size of area over which one measures other parameters.

Grain

Wood and Snell (1960, p. 1) defined grain as "the size of area over which the other factors are to be measured. It is dependent on the spacing of major ridges and valleys and thus indicates texture of topography." Grain was calculated by determining the local relief within concentric circles around a randomly-located point. Relief was plotted against diameter and, according to the authors, there will generally be a "knick point" in this curve—the diameter at this point will be the grain (G). Wood and Snell used diameter increments of one mile, and suggested that if there is no knick point, graphs for a number of sample points should be averaged. They noted that the method is not very precise, but believed that it was better than measuring parameters such as relief for standard arbitrary areas. Other parameters should be sampled over areas larger than or equal to the grain size in order to obtain representative values.

Texture

As noted above, texture is herein used in a general sense to refer to the shortest significant topographic wavelength. This should determine the spacing of sample points when the surface is digitized for computer use. The word "texture" has also been used for a specific geomorphometric parameter. Smith (1950) proposed a *texture ratio:*

$$T = N/P \qquad (1)$$

perimeter of the basin given in miles or fractions thereof" (p. 657). He "selected" the contour having the most crenulations. Smith found that the texture ratio was closely related to the drainage density.

Drainage density (D_d)

Drainage density, defined by Horton (1945, p. 283) as the total length of stream channels per unit area, represents a very important geomorphometric parameter. It has been found to be closely related to mean stream discharge (*cf.* Carlston 1963), mean annual precipitation (*cf.* Chorley and Morgan 1962), and sediment yield (Abrahams 1972). It has also been shown to increase with time on till plains exposed by deglaciation (Ruhe, 1952). Roberts and Klingeman (1972) found that the total length of flowing channels at a particular time is closely related to instantaneous stream discharge. Thus drainage density for *flowing* channels only will vary over short periods of time.

In a method analogous to Wentworth's (1930) method for slope estimation (see below), Carlston and Langbein (unpub. 1960, *cf.* McCoy 1971) and McCoy (1971) used traverse sampling to obtain a rapid estimate of drainage density. For a discussion of this method, see Mark (1974*b*). Other writers have used the numbers of intersections between the drainage net and traverse lines directly without attempting to convert them to drainage density (Wood and Snell 1957, 1959, 1960, Peltier 1962, Donahue 1972). A related method using a dot planimeter was proposed by Donahue (1974).

Another parameter very closely related to drainage density is the source density (D_s), the number of stream sources per unit area (*cf.* Mather 1972, p. 311). Both this and the preceeding parameter are very sensitive to possible map-to-map inconsistencies in the portrayal of the drainage net, and for this reason some writers have used the "extended drainage network" formed by extending streams as indicated by contour crenulations. This, however, introduces an element of subjectivity. The quality of the blue-line drainage net shown on some topographic maps from southern British Columbia was investigated and the results will be reported below.

Other texture measures

number of closed hilltop contours per unit area, here termed the *peak density* (D_p). Wood and Snell (1959) used this as one of their parameters for classifying terrain. King (1966) and Swan (1967) also used this measure. Using a related parameter, Ronca and Green (1970) studied the density and distribution of craters on the lunar surface.

Yet another way of characterizing surface texture is through an examination of ridges. Speight (1968) determined *ridginess*, the total length of ridge per unit area (analogous to drainage density) and *reticulation*, which was a measure of the size of "the largest connected network of crests that projected into a sample area" (p. 248). He also used modified two-dimensional vector analysis on ridge segments to measure the degree to which the ridges tended to be parallel. These and the other texture measures appear to be amenable to computerization.

Relief measures

The term *relief* is used to describe the vertical dimension or amplitude of topography. Evans (1972, p. 31—32) noted that the majority of relief measures depend upon the extreme values of the distribution of elevations, and would thus be sensitive to rather minor variations in estimations of these extrema. He therefore proposed that the standard deviation of altitudes would provide a more stable measure of the vertical variability of the terrain. He did observe that "the autocorrelation of altitude admittedly makes range less unreliable than it is for random variables, since on a continuous surface all intermediate values between the extremes must be represented" (p. 31), but nevertheless recommended use of the standard deviation. All of the other papers known to the writer have, however, used extreme values to characterize the vertical dimension.

Local relief (H)

For any finite area of a surface, the *local relief* is defined as the difference between the highest and lowest elevations occurring within that area. It is important to note that local relief is always defined with respect to some particular area, and perhaps for this reason has sometimes been termed the "relative relief"

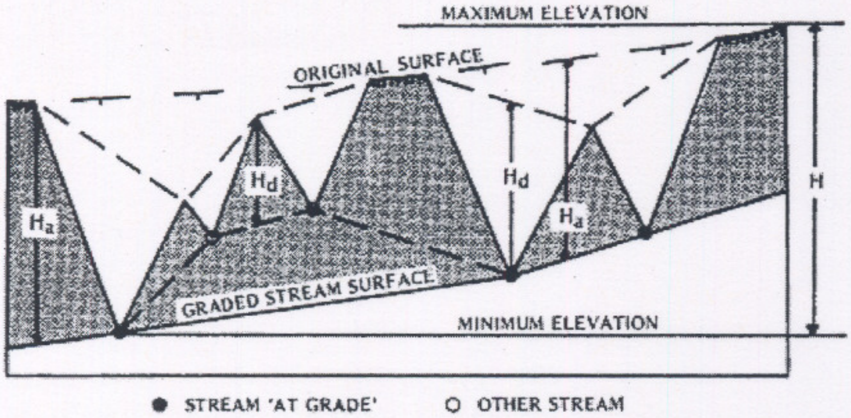

Fig. 1. Hypothetical topographic profile illustrating various relief measures. H is the local relief for the entire profile, H_a Glock's available relief, and H_d the drainage relief. Dury's "available relief" would be the *mean* height of the shaded portion.

introduced by Partsch (1911), who termed it the *Reliefenergie,* and was first used in the English language in 1935 in independent papers by Huggins and by Smith.

These works, as well as many others (see Mark 1974*a*, p. 31), determined local relief for arbitrarily-bounded terrain samples such as squares, circles, or latitude-longitude quadrangles. In most cases, the size of the sample area was arbitrary, although Trewartha and Smith (1941, p. 31) stated that "the size of the rectangle for which relief readings are made appears to need adjustment for the degree of coarseness or fineness of the relief pattern." They did not indicate how the appropriate size could be determined. Wood and Snell's (1960) "grain" method (see above) would be one solution. Wood and Snell (1957, 1959), Peltier (1962), and Evans (1972) compared the values of local relief determined over more than one size of area. Evans (1972, p. 30) pointed out that if the sample area "is so small (in relation to topographic wavelengths) that it is unlikely to contain a whole slope, 'relief' becomes simply a measure of gradient;" in order to make relief "as distinct and non-redundant a variable as possible" (p. 31), he recommended the use of "fairly large" sample areas. The areas should definitely be larger than the texture of the topography, and preferably larger than its grain. Data from Wood and Snell (1959, p. 9) support Evans' contention—they found that the correlation between relief and slope declined as the size of the area over which they were measured increased.

In all of the above examples, local relief was determined for arbitrarily-bounded areas;

Since the size of drainage basins varies, many workers have determined a dimensionless "relief ratio" or "relative relief number" by dividing the relief by some other linear dimension of the basin. The latter have included basin diameter (Maxwell 1960), basin perimeter (Melton 1957) and square root of basin area (Melton 1965).

Available relief (H_a)

The concept of *available relief* was introduced by Glock (1932), and his definition was rephrased by Johnson (1933, p. 295) to read: "Available relief is the vertical distance from the former position of an upland surface down to the position of adjacent graded streams." Johnson pointed out that this could only be determined where the original upland could be identified from remnants and where there were "graded" streams. The various relief concepts are illustrated in Fig. 1. Glock stressed the importance of available relief in determining the land profile but, as Johnson noted, other factors such as drainage density and slope must also be considered. In order to determine the average available relief, one would have to construct either by hand or with the computer both the "original" and "graded streamline" surface (see Pannekoek 1967) and to then divide the difference in the volumes under these surfaces by the area.

A different relief measure was discussed by Dury (1951), who unfortunately also used the term "available relief"; this was defined as "that part of the landscape which, standing higher than the floors of the main valleys, may be looked on as available for destruction defined the "mean available relief" as the *average* height of the land above the streamline surface, computed as the difference in volumes under the *actual* and streamline surfaces, divided by the area. This is clearly not the same as the available relief defined earlier. Dury (1951, p. 342—3) also discussed the "depth of dissection", which is identical to the Glock/Johnson concept of available relief.

Drainage relief (H_d)

Glock (1932, p. 75) and Johnson (1933, p. 301) also defined a measure called the *drainage relief*, the vertical distance between adjacent divides and streams (see Fig. 1). If in an area all the divides are remnants of an original upland surface and all the streams are "at grade", drainage relief will equal available relief; in contast to the latter, however, drainage relief can always be determined. Strahler (1958, p. 295) stated that "local relief, H, is a measure of vertical distance from stream to adjacent divide", but this will only be true if the sample areas upon which local relief is based are of an appropriate size. In Fig. 1, the area over which H is determined is relatively large and hence H exceeds H_d.

Applications of relief measures

Relief has commonly been used in a descriptive way (*e.g.* Smith, 1935) or to delimit physiographic regions (*e.g.* Huggins, 1935), both alone and in conjunction with other variables. Some studies have, however, related relief to landscape processes. Schumm (1954, 1963) found that sediment yield was closely related to the ratio of basin relief to basin diameter for some small drainage basins in the southwestern United States. Schumm (1956) also related sediment yields to relief and slope for some smaller basins in the Perth Amboy badlands. Maner (1958) investigated the relationships between sediment yield and a number of basin characteristics, and found that the relief:diameter ratio was most highly correlated with the dependent variable. Ahnert (1970) determined average basin relief as the mean of local relief values for 20 by 20 km squares spread over a number of large basins for which he had information on denudation rates. In the absence of stream incision, denudation relief, Ahnert determined theoretical curves for relief reduction as a function of time. He later (1972) related these results to theoretical models for slope processes.

It would appear that for both computational and geomorphic reasons, local relief for standardized sample areas represents the best single measure of the vertical dimension. It can be readily obtained from a computer representation of a surface.

Slope

Evans (1972, p. 36) stated that "slope is perhaps the most important aspect of surface form, since surfaces are formed completely of slopes, and slope angles control the gravitational force available for geomorphic work." Mathematically, the tangent of the slope angle ($\tan\alpha$) is the first derivative of altitude, and it is as a tangent or per cent slope that this surface parameter is generally reported. Strahler (1956) also mapped slope sine, which is proportional to the downslope component of the acceleration of gravity. Strahler's (1950, 1956) work suggested that slope tangents have a normal distribution; Speight (1971), however, found that for a number of areas, a log-normal distribution provided a better fit.

Unlike relief and most other parameters, which are only defined for finite subareas of a surface, slope is defined at every point as the slope of a plane tangent to the surface at that point. In practice, however, slope is generally measured over a finite distance, especially when data are obtained from a contour map. The size of area over which slope is measured will influence the values obtained, and the effect of recording intervals on slope values was discussed by Gerrard and Robinson (1971). Mean slope was generally much less sensitive to the recording interval than was maximum slope.

Average slope: Sampling methods

A method for estimating average slope proposed by Wentworth (1930) has been widely applied. The number (N) of intersections between a set of traverse lines and the contours in the sample area is counted, and the total length of the traverse lines (L) is measured

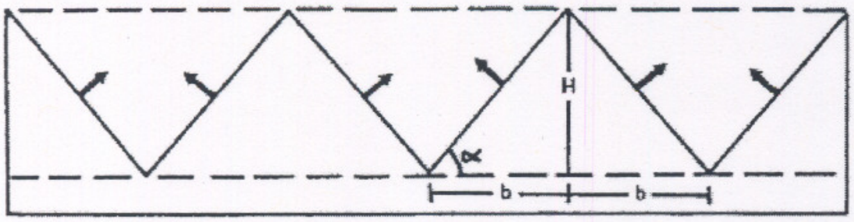

Fig. 2. Diagrammatic topographic profile illustrating the relationships among relief, slope, and roughness (see eqs. 4, 9, and 10).

$$\tan \alpha = I(N/L)/0.6366 \qquad (2)$$

where I is the contour interval in the same units as L. Wentworth presented the formula for use with L in miles and I in feet as:

$$\tan \alpha = I(N/L)/3361 \qquad (3)$$

The method gives the mean slope for an area, but has been used to construct slope isopleth maps by assigning the area's slope to a point at its centre (*cf.* Smith 1939, Calef and Newcomb 1953, Griffiths 1964).

Other authors have used the number of contour intersections per length of traverse directly, without converting to actual slope values. Wood and Snell (1957, 1960) used the "contour count" as a "measure of slope" (1957, p. 1), but in their 1959 paper converted to slope using Wentworth's formula. Zakrzewska (1963) determined the "roughness" as the number of contour intersections with the circumference of a circle.

In direct computer applications, a number of writers (*cf.* Monmonier *et al.* 1966, Park *et al.* 1971) have determined surface slope from digitized contour data. Sharpnack and Akin (1969) computed slope and aspect from a regular grid.

Griffiths (1964) compared the "subjective method" (essentially the Raisz and Henry, 1937, approach), Wentworth's method, and "point sampling". He concluded that Wentworth's method was the most accurate, and that the point sampling method produced "comparable" results with less effort.

Other slope parameters

Another slope parameter is the rate of change of slope, termed the "local convexity" by Evans (1972, p. 41). Mathematically, this is the second derivative of altitude, or the first derivative of slope. Convexity can be separated ed" by fitting quadratic surfaces to 3 by 3 sections of a regular grid. Convexity would then be the second derivative of the resulting quadratic equation. Speight (1968) examined both rate of change of slope (which he termed "slope gradient") and contour curvature. It is also possible to determine higher derivatives of altitude, but the possible physical meaning of such derivatives is obscure.

Closely related to mean slope is Strahler's (1958) *ruggedness number,* defined as HD_d as a result of dimensional analysis. In the case of a two-dimensional profile, the relationships among relief, drainage density, and slope can be easily shown. In Fig. 2, H is the relief and b half the distance between channels, which equals half the inverse of D_d. One thus has the mean slope given by:

$$\tan \alpha = H/b = 2HD_d \qquad (4)$$

or twice the ruggedness number. Strahler (p. 295 also introduced average slope into the ruggedness number, producing the *geometry number:*

$$HD_d/\tan \alpha \qquad (5)$$

If H is reasonable estimate of the drainage relief and if the two-dimensional case can be extended to three dimensions, this geometry number should equal 0.5. Strahler found that while drainage density for this test basins ranged over two orders of magnitude, the geometry number remained between 0.4 and 1.0. As such, this parameter is probably of little value.

Application of slope measures

As in the case of relief, slope has been widely used in descriptive work, in physiographic classification, and in military work related to vehicle trafficability. Slope angle is a result of form represents an important "subdiscipline" of geomorphology (*cf.* Institute of British Geographers 1971, Carson and Kirby 1972, Young 1972).

Dispersion of slope magnitude and orientation

In addition to slope steepness, slope aspect or direction may be considered, either separately or together with slope angle. Evans (1972, p. 41) proposed that the combined analysis of slope magnitude and orientation would produce "undesirable hybrid results; it is better to separate variability in gradient from variability in aspect." If this is done, the aspect data should be analyzed using two-dimensional vector analysis (*cf.* Curray 1956). While such separation may be desirable in some cases, the distribution of orthogonals to the land surface (which summarize both types of information) *is* essentially three-dimensional, and its analysis as such would seem to be appropriate.

Chapman (1952) presented a potentially useful method for examining slope steepness and aspect. Both the aspect (orientation) and slope (dip) of the land surface were determined for a sample of points on a regular grid. The points were then plotted on a Schmidt net and contoured in the same way as are other orientation data in the earth sciences. Chapman suggested that these diagrams would probably be useful in relating slopes to structure or to effects of glacier movement, and Newell (1970) succesfully used the technique in this context. One of the computer programs presented by Hobson (1967) represents a logical extension of this work, treating perpendiculars to slope facets as vectors and applying well-established mathematical approaches to the analysis of three-dimensional orientation data (*cf.* Fisher 1953, Steinmetz 1962). Unit vectors orthogonal to triangular facets formed by inserting diagonals into a regular grid were summed and the length of the vector sum (R) was determined. Hobson then calculated k as:

$$k = (N-1)/(N-R) \qquad (6)$$

As a surface approaches planarity, the vectors will become parallel, R will approach N (the ness of zero, and thus the inverse of k would represent a more "reasonable" roughness measure. Since Hobson's method was based on a regular grid, all triangles have the same horizontal area and similar true areas, and hence the use of unit vectors is not unreasonable. If based upon irregularly-distributed surface-specific points, however, there may be a considerable variation in triangle size. It would seem appropriate to weight the vector orthogonal to each triangle by the triangle's true area. If this is done, however, k and its inverse cannot be determined through eq. (6). Some manipulation of that equation gives:

$$\frac{1}{k} = \left[\frac{N}{N-1}\right]\left[1-\frac{R}{N}\right] \cong \frac{100-L(\%)}{100} \quad \text{(for large } N\text{)} \qquad (7)$$

where $L(\%)$, is $100(R/N)$, the vector strength in per cent. For weighted vectorial analysis, L is defined as 100 times the weighted vector sum divided by the sum of the weights. It is herein proposed that the best measure of vector dispersion roughness is the *roughness factor IR*, defined by:

$$IR = 100-(L)\% \qquad (8)$$

In the case of unit vectors and large N, IR will approximately equal 100 times the inverse of k.

As in the case of slope, the roughness factor can be related to relief and texture in the two-dimensional case through reference to Fig. 2. For IR, the vertical component of each orthogonal vector will equal $\cos\alpha$, while the horizontal components will cancel out, leaving:

$$IR = 100\,(1-\cos\alpha) \qquad (9)$$

Substituting the value for $\cos\alpha$ gives:

$$IR = 100\left(1-\frac{b}{\sqrt{H^2+b^2}}\right) \qquad (10)$$

In Fig. 3, IR (as estimated from k) is plotted against H for 25 samples studied by Turner and Miles (1967) and for six others analyzed in this study. Curves of the form given by eq. (10) for various values of b have been plotted in Fig. 3. These have been fitted "by eye" to the groups of points for each of the six scales represented. It appears that each scale has a reasonably consistent "characteristic

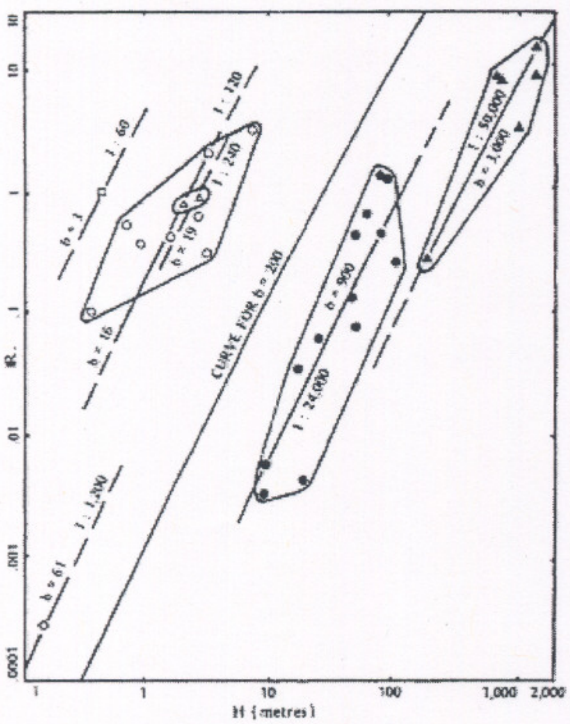

Fig. 3. Relationship between local relief (H) and the roughness factor (IR). Open symbols represent micro-terrain from Turner and Miles (1967); solid symbols are macro-terrain (circles from Turner and Miles; triangles from this study). Curves are based on eq. (10). Map scales are indicated on the curves.

Hypsometry

Clarke (1966 p. 237) definied hypsometry as "the measurement of the interrelationships of area and altitude." Evans (1972, p. 42—48) reviewed this concept uner the heading: "Regional convexity (dissection, aeration)." Most of these measures, which describe aspects of the distribution of landmass with elevation, are based upon the hypsometric curve.

The Hypsometric curve and its variations

Monkhouse and Wilkinson (1952, p. 112—115) noted that there are three common sorts of graphs used to report hypsometric data. These are:

(a) the area-height curve;

(b) the hypsometric (or hypsographic) curve, sometimes called the absolute hypsometric curve;

(c) the percentage hypsometric curve.

by convention, elevation is plotted on the y-axis. If relative area is used, the diagram is a plot of the probability density function for the heights. The relative frequencies of various elevations are generally more easily seen on this type of curve than on the others.

The absolute hypsometric curve is a graph of the absolute or relative area above a certain elevation plotted against that elevation, and is essentially a cumulative frequency diagram for the elevations. Once again, elevation is conventionally plotted on the y-axis.

The third and most widely used form of curve is the relative or percentage hypsometric curve, often termed simply the *hypsometric curve*. It plots relative area above a height against *relative* height, and is the graph of the hypsometric function, here termed $a(h)$, where h (the relative height) is defined by:

$$h = \frac{z - z_{min}}{z_{max} - z_{min}} \qquad (11)$$

where z is the actual elevation and z_{max} and z_{min} are the highest and lowest elevations, respectively, within the study area. As in the previous cases, h is conventionally plotted on the y-axis. It is this form of the hypsometric curve and function upon which some important terrain parameters are based.

The hypsometric integral (HI)

The most widely used parameter based on the hypsometric curve is the *hypsometric integral*, here designated *HI*. This parameter, as defined by Strahler (1952, p. 1121), is given by:

$$HI = \int_0^1 a(h)\, dh \qquad (12)$$

Strahler pointed out that geometrically, this value is equal to the ratio of the volume between the land surface and a plane passing through z_{min} to the volume of a "reference solid" bounded by the perimeter of the area and planes through z_{max} and z_{min}. Graphically, HI can be determined by measuring the area under the relative hypsometric curve.

Pike and Wilson (1971) proved that the elevation-relief ratio (E) of Wood and Snell (1960) is mathematically equal to the hypsometric integral. The former is defined by:

where $\bar{z}$ is the mean elevation. From eqs. (11) and (13), it can be seen that E is just the mean relative height ($\bar{h}$). Evans (1972, p. 42) pointed out that this same parameter was used much earlier by Péguy (1942, p. 462), and termed the "coefficient of relative massiveness" by Merlin (1965). While Strahler's (1952) method for determining *HI* involves much laborious use of a planimeter to determine inter-contour areas, E can be determined much more quickly, with the mean elevation determined from a sample of points. Pike and Wilson (1971, p. 1081) stated that "experience has shown that a sample of 40 to 50 elevations will ensure accuracy of E to, on the average, 0.01, the value to which area-altitude parameters customarily are read." It is important that the maximum and minimum elevations are determined from an inspection of the entire sub-area; gross errors in E can result if the highest and lowest grid values are used (Mark, 1974*a*, p. 87). Evans (1972, p. 58), however, used only grid values to estimate the hypsometric integral for sub-matrices ranging from 3 by 3 to 47 by 47. For the smaller sub-matrices at least, Evans' estimates of *HI* are probably in serious error.

Other methods for approximating the hypsometric integral or curve have been proposed. Haan and Johnson (1966) suggested that the elevations of a sample of randomly-located points could be used to construct hypsometric curves, with considerable saving in time. Chorley and Morley (1959) proposed that the hypsometric integral could be estimated by approximating a drainage basin by a simple geometric form. Turner and Miles (1967) used a computer program to interpolate a dense regular grid from a sample of points; numbers of grid points falling within altitudinal bands were used in producing hypsometric curves. They found that their method produced results closer to planimetered values than did the Chlorley and Morley approach. It would seem that the elevation-relief ratio is a more accurate and more easily applied approximation to the hypsometric integral than are the above. Furthermore, E can be determined for arbitrarily-bounded areas (*c.f.* Wood and Snell 1960, Pike and Wilson 1971), while the

Other parameters related to hypsometry

A number of other parameters have been derived from the hypsometric curve. Strahler (1952, p. 1130) noted that most hypsometric curves show a characteristic "s-shape", and proposed a parameter to indicate the sinuosity of the curve. Low values of this parameter indicated very sinuous curves. Evans (1972, p. 47—48) found a strong correlation between *HI* and the skewness of the distribution of elevations in cases having the same sinuosity. For any constant value of *HI*, higher skewness was associated with lower sinuosity. Tanner (1959, 1960) suggested that the skewness and kurtosis of the height distribution function (essentially the hypsometric function) could be used to "characterize various geomorphic regions" (1960, p. 1525). Examination of Tanner's diagrams seems to confirm Evans' result that skewness is closely related to the hypsometric integral, and also suggests that Strahler's sinuosity parameter is closely related to kurtosis. The latter have not (to writer's knowledge) been investigated in detail or related to other geomorphometric measures.

Gassman and Gutersohn (1947) determined a parameter called the *Kotenstreuung*. For computation, this has been shown to equal the standard deviation of the elevations, and was derived from the absolute hypsometric function. They also determined the *Relieffactor*, which equals twice the *Kotenstreuung* divided by the local relief. This is twice the standard deviation of the *relative* hypsometric function. Gassman and Gutersohn also determined the mean elevation by using the hypsometric integral, "reversing" the use of the elevation-relief ratio proposed above; this method of determining the mean elevation was employed earlier by de Martonne (1941).

In addition to those related to the hypsometric curve, other parameters have been proposed to characterize the relationship between area and altitude, sometimes also including slope. None of these have been as widely used as the hypsometric integral; since many of these have been reviewed by Clarke (1966, p. 243—248) and by Evans (1972, p. 44—45), they will not be reviewed herein.

Table 1. Variables included in correlation analysis.

Variable Number	Symbol	Name
1	D_d	Drainage density
2	N/L	Drainage net intersections
3	D_s	Source Density
4	D_p	Peak Density
5	H	Local relief
6	H^*	Grid estimate of local relief
7	$\tan \alpha$	Average slope tangent
8	HI	Hypsometric integral
9	HI^*	Grid estimate of HI
10	$\bar{z}$	Mean elevation
11	p	Mean annual precipitation
12	t	Year of map publication

or in physiographic classification. Only the hypsometric integral, however, has been related to geomorphic processes. Strahler (1952, p. 1130) proposed that the value of the hypsometric integral reflects the "stage" of landscape development. Those areas having *HI* values above 0.6 were considered to be in a "youthful" or inequilibrium phase, while drainage basins in equilibrium should have hypsometric integrals between 0.6 and 0.35. Values below 0.35 were thought to characterize a transitory "monadnock phase" in landscape development. Strahler (1957, p. 918—920) listed a number of works between 1952 and 1956 which used this parameter; none of these found any relationship between *HI* and various hydrologic or sediment yield measures.

Relationships among variables

In order to investigate empirically the relationships among terrain and related parameters, linear correlation coefficients were computed among ten terrain measures and two other variables listed in Table 1. The parameters were manually determined or estimated for each of 42 7 by 7 km terrain samples from 1:50,000 scale topographic maps of southern British Columbia. The size of the sample areas was selected arbitrarily, and the samples were located according to a stratified random sampling design. For further details of the sampling procedure, see Mark (1974*a*). Those parameters based on a grid (see Table 1) used a 7 by 7 grid with a 1 km spacing. Table 2 contains all correlation coefficients which were statistically significant at the 95 % level. These were examined using the approach outlined by Melton (1958); Fig. 4 illustrates the three isolated correlation sets which form the cores of three variable systems, namely "drainage texture", "relief", and "hypsometry". Peak density (D_p) was not significantly correlated with any other variable.

The drainage parameters were based upon the "blue line networks" of streams printed on the maps used as a data source. The writer had observed during the data collection that some of the older maps appeared to have higher drainage densities than newer ones. Because many of the older maps were coastal, it was thought that the variation might be physical rather than cartographic. For this reason, both mean annual precipitation and year of map publication were included in the correlation analysis. As shown in Fig. 4, map year was inversely correlated with the drainage parameters. There was no significant correlation between map year and mean annual pre-

Table 2. Statistically significant (95 per cent level) linear correlation coefficients among the variables listed in table 1.

D_d	N/L	D_s	D_p	H	H^*	$\tan \alpha$	HI	HI^*	$\bar{z}$	p	t	
—	0.990	0.921								0.473	−.496	D_d
	—	0.927								0.468	−.485	N/L
		—								0.554	−.479	D_s
			—									D_p
				—	0.987	0.824			0.419			H
					—	0.797			0.364			H^*
						—		0.323		0.602		$\tan \alpha$
							—	0.887	0.364			HI
								—				HI^*

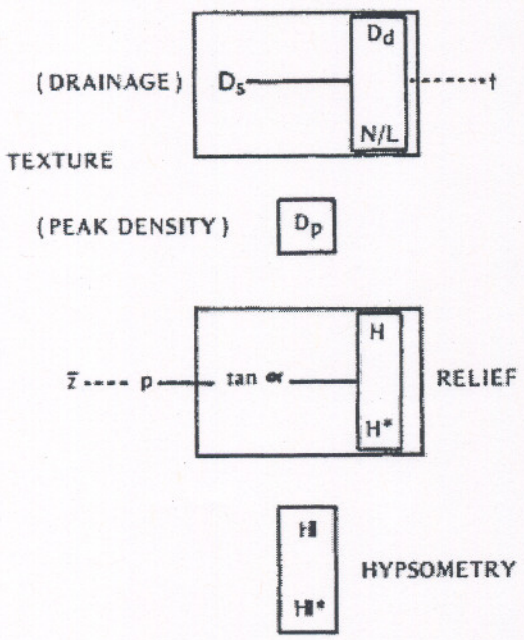

Fig. 4. Correlation structure among twelve terrain and related parameters, constructed in the manner proposed by Melton (1958). The outer boxes enclose isolated correlation sets; dotted lines indicate inverse correlations.

cipitation, suggesting that variations in drainage parameters were indeed at least in part cartographic.

Conclusions

In review, the most fundamental concepts of geomorphometry are the basic horizontal and vertical scales of the topography. Horizontal variations are encompassed by the concepts of grain (largest significant wavelength) and texture (shortest significant wavelength). Even if these are not explicitly investigated, these concepts will be implicit in a computer analysis of topography, texture in the sampling density and grain in the area over which other parameters are estimated. The correlation analysis indicated that drainage measures and peak density represent different aspects of texture.

Vertical scale is generally termed relief; this terrain concept is probably best represented by the local relief (*H*), the most widely employed relief measure. The relationships between horizontal and vertical scales are examined through the mean slope (tan α), which est. The three-dimensional interaction of slope steepness and aspect is represented by the roughness factor (*IR*) introduced in this study; the relationships of this to the other variables were not examined empirically because of the time which would have been required to prepare the forty-two topographic samples for computer analysis.

Relatively independent from horizontal and vertical scales is the distribution of mass within the vertical range of the topography. This concept is best represented by the hypsometric integral (*HI*).

While there may be some redundancy among the parameters noted, it is believed by the writer that with the possible exception of local convexity, all important terrain information is contained within the above measures. In the second part of this study (Mark, 1975), computer terrain storage systems are reviewed, and the relative merits of two of these systems for the estimation of some of the above parameters is discussed.

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