# (Uncertainty and) The Firm in General Equilibrium Theory

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# (UNCERTAINTY AND) THE FIRM IN GENERAL EQUILIBRIUM THEORY*

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The firm fits into general equilibrium theory as a balloon fits into an envelope: flattened out! Try with a blown-up balloon: the envelope may tear, or fly away: at best, it will be hard to seal and impossible to mail.... Instead, burst the balloon flat, and everything becomes easy. Similarly with the firm and general equilibrium – though the analogy requires a word of explanation.

General equilibrium theory – GET for short – has two attributes. First, it defines *clearly* the boundary between economic analysis and the exogenous primitive data or assumptions from which it proceeds; that is, it defines a precise, self-contained 'model'. Second, it verifies the overall *consistency* of the economic analysis. A natural step in verifying overall consistency is to exhibit sufficient conditions for the *existence* of the proposed solutions, or 'equilibria'. This step is usually amenable to mathematical reasoning.

Still, I do not mean to *identify* general equilibrium theory with that potent cocktail of economics and mathematics known as mathematical economics. (To some, mathematical economics is merely a pleonasm; to others, it is a branch of mathematical pornography; the word cocktail, with its element of pornographic pleonasm, is purposedly neutral.) Work in mathematical economics lacking the GET – attributes is abundant. Conversely, let me remind you of our friend Harry Johnson: He was a beautiful member of our GET-set, for he was skilful at integrating partial contributions into consistent general pictures; yet he needed little algebra, because he was a master at the declining art of expressing complex rigorous arguments in literate English.

The two attributes of General Equilibrium Theory stand out, for instance, in the classic work *Theory of Value* – hereafter referred to as TV (not inappropriately, given the recent successes of its author as a TV-star). In *Theory of Value*, an 'economy' is defined by a set of commodities; a set of consumers, individually described by their needs and abilities (consumption sets), their initial assets, and their tastes (preferences); and a set of producers of firms, individually described by their technological possibilities (production sets). These are the primitive data, which the economist treats as exogenous and does not seek to explain. Basically, they correspond to the *opportunities* and *motivations* of all agents – a structure which is even clearer in the 'abstract economies' approach; see Shafer and Sonnenschein (1975).

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The point I wish to emphasise is that, in *Theory of Value* and related works, a firm is described *only* and *fully* by its production set, or technology; it has no idiosyncratic behavioural attributes whatever. The common motivation or decision criterion of all firms, namely *profit maximisation at given prices*, reduces management to profit calculations and comparisons. The arithmetic may be laborious – still, no behavioural identity, or managerial initiative is involved. The TV-firm is an anonymous computer, it has the behavioural depth of a flattened out balloon, it is the negation of managerial skill and management education. (I once read about 'the doubtless mythical review of *Lady Chatterley's Lover* in *Country Life* where the writer supposedly complained that much interesting material about being a gamekeeper was constantly interrupted by irrelevant personal experience' (Nixon 1982). The TV-firm is like a platonic gamekeeper, endowed with a technology, but not susceptible of personal motivation and experience.)

This representation has been challenged, and various partial equilibrium contributions aim at extending it. Most notable among these is the Behavioural Theory of the Firm of Simon (1979), Cyert and March (1963). I shall not survey these contributions, but rather concentrate on the one area where formal extension is deliberately sought in a general equilibrium framework, namely uncertainty with incomplete markets. (It is also the one area with which I have some familiarity.)

Uncertainty is an intimate dimension of our daily lives. For some, it is the zest of life. Without uncertainty, the distinction between the present and the future is blurred; there are no surprises and no anticipations, hence no thrills; there is no scope for achievement, hence no rewards; and love, which always entails risks as well as the joy of discovery, looses its sharp edge. Yet, for others, uncertainty is the curse of life. It is so for those who feel threatened with loss of life or individual freedom, who have no assured shelter or subsistance, who lack job security and fear unemployment.

Uncertainty is thus an intimate dimension of economics as well. Decisions of households, firms or policy makers seldom entail fully deterministic consequences; uncertainty is 'generic'. (Still, many titles in economics include the precision 'under uncertainty', not even parenthetical; whereas it should be natural to include the warning 'under certainty', when appropriate.)

Uncertainty has been introduced formally into the model of *Theory of Value* by recognising that the primitive data or *environment* – in particular resources, tastes and technology – are not known and given, but are part of the unfolding history of the world. As of any future date, the world is apt to find itself in any one of several alternative, mutually exclusive *states* – each of which corresponds, among other things, to a history up to that date of resources, tastes and technology. Recognising that 'the environment is state-dependent' draws a credible boundary between the exogenous sources of uncertainty and economic analysis; we should be grateful to Savage (1953, 1954) and Arrow (1953) in particular for introducing that approach, which is more satisfactory than earlier formulations in terms of probability distributions of economic variables (like prices or incomes).

This is a model of 'Technological Uncertainty'. Recently, economists have tackled the more subtle problem of 'Information', about which you will hear tomorrow from the horse's mouth, if I may refer colloquially to my thoroughbred friend of a younger generation (Stiglitz, 1984). (See also Hirshleifer and Riley (1979) who elaborate on the distinction, and relation, between uncertainty and information.)

A model where the environment is state-dependent is amenable to the same formal analysis as a model where the environment is given, if one is willing to assume the existence of a complete set of insurance markets, one for each physical commodity contingent on each state of the environment. In that case, business decisions reduce again to arithmetic, because each production plan has a well-defined present value on the insurance markets. In particular, a firm contemplating a new investment could simultaneously protect itself against demand uncertainties by selling its output at each date on a futures market, and against supply uncertainties by purchasing insurance against output deficiencies, whether they be due to machine breakdowns, low labour productivity or mismanagement.

This is clearly an excessive idealisation. Existing insurance opportunities are limited by transaction costs, moral hazard, adverse selection and the like[1]. Incomplete markets are the rule, and firms come to life as they face the non-insurable uncertainties of history in the making. At once, they become concerned with forecasting and risk taking, which involve more than arithmetic (fortunately, say the econometricians and decision theorists). But as firms come to life, they fit less easily into the envelope of general equilibrium theory: there is today no formal description of 'live' firms which is both generally accepted and suitable for the purposes of general equilibrium theory.

This is unfortunate, because it hampers the development of general equilibrium theory, in all the directions where business uncertainties are essential; and these include investing, pricing, hiring, colluding, etc,.... At the same time, progress seems at hand, to the extent that most of the descriptions which have been used can be suitably reconciled and integrated, as I hope to show now.

## I

A simple model has proved very useful to clarify these issues, namely the temporary general equilibrium model of a stock-market economy, introduced by Diamond (1967) in the spirit of Arrow's (1953) initial suggestion. (This is a streamlined model, as could be expected. Indeed, models basically play the same role in economics as in fashion: they provide an articulated frame on which to show off your material to advantage...; a useful role, but fraught with the dangers that the designer may get carried away by his personal inclination for the model, while the customers may forget that the model is more streamlined than reality.)

[1] Note also that the set of potential buyers on a long-term futures market may include as yet unborn consumers...

There are two periods, the present (o) and the future. There are finitely many ($S$) possible states, indexed $s=1,\dots,S$. The true state is unknown in the present, but will be known in the future. There is a given set $J$ of firms, indexed $j=1,\dots,J$. A production plan for a firm $j$ is an $(S+1)$-dimensional vector $\mathbf{y}^j=(y_0^j,y_1^j,\dots,y_S^j)$, consisting of an input level $y_0^j$ in the present and a vector of state-dependent outputs $y_1^j,\dots,y_S^j$ in the future. The set of technologically feasible production plans is a given convex set $Y^j$ in $R_- \times R_+^S$. The problem of the firm is to choose a production plan $\mathbf{y}^j$ in $Y^j$.

There are no markets for contingent claims. But there is a stock market, where shares of stock in the various firms are traded at prices $\mathbf{p}=(p_1,\dots,p_J)$. The price $p_j$ is the market value of firm $j$, i.e. the price of a 100% share of that firm.

There is a given set $I$ of consumers, indexed $i=1,\dots,I$. A consumption plan for consumer $i$ is an $(S+1)$-dimensional vector $\mathbf{x}^i=(x_0^i,x_1^i,\dots,x_S^i)$ in $R_+^{S+1}$, specifying a current consumption level $x_0^i$ and a future consumption level $x_s^i$ in state $s$, $s=1,\dots,S$. Consumer $i$ has well-defined, continuous, convex, monotone preferences over consumption plans. These preferences reflect simultaneously time preferences, probability beliefs, attitudes to risk and interactions between states and consumption. They are represented by the utility function $u^i(x_0^i,x_1^i,\dots,x_S^i)$, here assumed differentiable for convenience. (See, e.g. Hirshleifer (1966) or Drèze (1982) for details.)

Also, each consumer $i$ has an initial endowment of consumption goods $\boldsymbol{\omega}^i=(\omega_0^i,\omega_1^i,\dots,\omega_S^i)$ in $R_+^{S+1}$, and of shares of stock in the firms $\bar{\boldsymbol{\theta}}^i=(\bar{\theta}^{i1},\dots,\bar{\theta}^{iJ})$, where $\bar{\theta}^{ij}\geqslant 0$ for all $i,j$ and $\sum_i\bar{\theta}^{ij}=1$ for all $j$.

With no markets for contingent claims, the only possibility for consumers to transfer resources between the present and the future, or between different states in the future, is to modify their initial portfolios of shares by trading on the stock market. These transactions, leading to the new portfolio $\boldsymbol{\theta}^i=(\theta^{i1},\dots,\theta^{iJ})$ in $R_+^J$, determine the final consumption levels[1]

$$x_0^i=\omega_0^i+\sum_j p_j(\bar{\theta}^{ij}-\theta^{ij})+\sum_j\theta^{ij}y^j\geqslant 0; \tag{1}$$

$$x_s^i=\omega_s^i+\sum_j\theta^{ij}y_s^j,\ s=1,\dots,S. \tag{2}$$

Condition (1) is a budget constraint, stating that current consumption is equal to the value of initial resources, $\omega_0^i+\sum_j p_j\bar{\theta}^{ij}$, minus the cost of the final portfolio, $\sum_j p_j\theta^{ij}$, and $i$'s share of the initial dividends (or fund raising) in the firms where he is a shareholder. Condition (2) states that future consumption is the sum of initial resources $\omega_s^i$ and the dividends accruing to the portfolio, $\sum_j\theta^{ij}y_s^j$.

The problem of consumer $i$ is to choose the portfolio $\theta^i$ that maximises $u^i(x_0^i,x_1^i,\dots,x_S^i)$ subject to the current budget constraint ($x_0^i\geqslant 0$), with $\mathbf{x}^i$ defined by (1)–(2)[2].

[1] In addition, $1\geqslant\theta^{ij}\geqslant 0$ must hold for each $j$; no inequality constraint is stated on $x_s^i$, $s=1,\dots,S$, because $\omega_s^i$ and $y_s^j$ (all $j$) are non-negative by assumption.

[2] In view of the equalities (1)–(2), consumption is uniquely determined by the portfolio choice, for given production plans of the firms.

An *equilibrium of the stock-market economy* consists of a price vector $\mathbf{p}$, a set of feasible portfolios $(\boldsymbol{\theta}^i)_{i\in I}$ and a set of feasible production plans $(\mathbf{y}^j)_{j\in J}$, such that:

(*i*) The stock market clears: for all $j$, $\sum_j \theta^{ij} = 1$;

(*ii*) each consumer is in equilibrium, given the prices $\mathbf{p}$ and the production plans $(\mathbf{y}^j)$: $\boldsymbol{\theta}^i$ maximises $u^i(\mathbf{x}^i)$ subject to (1) and (2);

(*iii*) each firm is in equilibrium: $\mathbf{y}^j$ is the best production plan in $Y^j$ – *in a sense that remains to be defined.*

When a business firm is unable to sell its production plan on insurance markets, it does not have well-defined profits; rather, a production plan induces a state-distribution of gross profits, $(y_1^j, \ldots, y_S^j)$ in the simple model under review. The question then arises: *How does a firm choose among alternative state-distributions of profits*? (For instance, how did McGraw Hill decide that it was worth their while to acquire DRI for a hundred million dollars – presumably without the benefit of DRI model simulations to guide the decision? How did General Motors recently decide how much to bid for Jaguar? How will tomorrow A. T. and T. decide how much to bid for the British Post Office, when it is put up for sale? How did yesterday that French firm specialised in industrial conditioning and packaging decide to diversify into services by opening a chain of funeral parlours?)

The question arises because firms, unlike consumers, are not human beings whose preferences could be introduced as primitive data. Economists have come to accept that a person's attitude towards risks is part of that person's identity, together with her attitude towards tea bags, rock music or central heating. But not so with business firms, which have no personal tastes for what they produce, and no visceral reactions to uncertainty[1].

Thus, risk preferences belong with human agents. In Diamond's model, business profits ultimately accrue to consumers as dividends (or capital gains). The risk preferences of consumers manifest themselves through portfolio choices, and are reflected in stock prices. In turn, the prices of shares on the stock market act as a partial substitute for insurance markets, in assigning a well-defined aggregate value to the production plan of each firm. This raises the more specific question: *Do the preferences of shareholders, and the prices of shares on the stock market, influence the choices of firms among alternative state distributions of profits*?

These questions have occupied theorists of uncertainty, and of finance, over the past 15 years. In a survey two years ago, published in a book with the unlikely title *Current Developments in the Interface* (Drèze, 1982) and soon to be reprinted in a volume of essays on *Economic Decisions under Uncertainty* (Drèze, 1985a), I found it helpful to classify their contributions in three 'nested' groups, as follows. (See also Grossman and Stiglitz, 1980; or Milne and Starrett, 1981.)

[1] This issue does not arise under complete markets because each production plan has a well-defined insurance value, so that profit maximisation is a well-defined, universal criterion.

II

It was noted above that the stock market assigns a well-defined aggregate value to the production plan of a firm. This value is often referred to as 'the market value of the firm'. By analogy with financial decisions, production decisions are sometimes presented (in textbooks or the oral tradition) as *maximising the market value of the firm.* This is the first group or approach.

In order for this criterion to be operational, a firm should know whether a given modification to its production plan will increase or decrease market value. If that information is to be found in stock-market prices, there must exist some portfolio of shares, whose net value is indicative of the incremental value of the proposed modification. Special assumptions are needed to prove the existence of market portfolios conveying the necessary information, for all possible modifications of a firm's plan. The assumptions underlying the *Capital Asset Pricing Model* are sufficient – but highly restrictive; the more general assumptions underlying the *Arbitrage Pricing Model* are not sufficient. Diamond's assumption of multiplicative uncertainty, which corresponds to the *Risk Classes* of Modigliani and Miller (1958), is also sufficient but equally restrictive. Both approaches are special cases of the so-called *Spanning Theory*, which applies when the production set of each firm is contained in a space spanned by the production plans of existing firms[1].

That special, restrictive assumptions are needed, to render the market value criterion operational, should not surprise us. Indeed, to say the market value is an operational criterion is to say that business management is again a matter of arithmetic, in contradiction to the observation that firms are directly concerned with forecasting and risk taking.

Recognising that stock prices (market values) do not always convey the information needed to guide production and investment decisions, a second group of theorists (namely, Grossman and Hart in 1979 and myself in 1974*b*) have investigated the possibility that stockholders' preferences may guide these decisions directly. (When that approach is presented to a Business School audience, it is typically greeted with sneers or laughter; fortunately, 'le ridicule ne tue pas'.)

*When spanning prevails*, all consumers holding portfolios which are optimal from the viewpoint of their consumption preferences (given the production plans of the firms and the stock-market prices) will unanimously approve production decisions maximising market value. *In the absence of spanning*, there is room for disagreement – for example between shareholders favouring more venturesome decisions, and those favouring more conservative decisions. It seems natural to require that business firms should at least respect *unanimous* wishes of their shareholders.

In Diamond's model, shareholder $i$ prefers production plan $\hat{\mathbf{y}}^j$ over $\mathbf{y}^j$ if and

[1] Technically, the Capital Asset Pricing Model has the 'spanning property' in the space of the two 'characteristics' (namely, mean and variance) in terms of which preferences can be represented. See Drèze and Hagen (1978) for details.

only if his resulting consumption has higher utility, that is if and only if[1]

$$u^i[\mathbf{x}^i + \theta^{ij}(\hat{\mathbf{y}}^j - \mathbf{y}^j)] > u^i(\mathbf{x}^i). \tag{3}$$

This leads to a criterion of *Nash-Pareto optimality*: the production plan $\mathbf{y}^j$ is best for firm $j$, given its ownership $\boldsymbol{\theta}^j = (\theta^{1j}, \ldots, \theta^{Ij})$, if there does not exist an alternative plan $\hat{\mathbf{y}}^j$ in $Y^j$ preferred to $\mathbf{y}^j$ by all shareholders, that is, by all consumers $i$ with $\theta^{ij} > 0$; (more generally, preferred by some shareholders and deemed indifferent by the remaining ones).

The Pareto aspect (for the owners of a given firm) is clear; the Nash aspect comes from the feature that the portfolios of consumers, and the production plans of other firms, are treated as given and fixed in the definition. As a consequence, Nash-Pareto optimality does not entail full Pareto optimality: it could be that simultaneous changes in the production decisions of several firms, or simultaneous changes in portfolios and in production decisions, would lead to Pareto-superior allocations; examples are given in Drèze (1974 *b*), where it is shown that the set of feasible allocations in this economy is in general *not* convex.

The Nash-Pareto criterion just defined has two attractive features. First, it preserves the consistency of decentralised decisions. As we shall see below, under standard (TV) assumptions, *there exist stock prices, portfolios and production plans defining an equilibrium of the stock-market economy, where the production plan of each firm is a Nash-Pareto optimum* for its own shareholders.

Second, the Nash-Pareto criterion is consistent with market value maximisation, under either complete or partial spanning. That is, whenever information contained in stock prices reveals that a potential modification of firm $j$'s production plan would increase market value, there will be unanimous approval of the modification by the shareholders. This also implies that formal elicitation of shareholders' preferences will be superfluous, in that case.

On the other hand, the Nash-Pareto criterion has two drawbacks. First, it only establishes a very partial ordering among production decisions, leaving many issues undecided. Because most decision situations call for definite choices, the criterion must be supplemented by a specification of how undecided issues are resolved. Second, consultation of shareholders is unwieldy, time consuming and costly. The relevance of this criterion thus seems limited to major, infrequent decisions (for instance, fixed investments financed by equity issues).

These two drawbacks explain why many authors assume outright the existence of a *Utility Function of the Firm*, defined *exogenously* over state-distributions of profits. That assumption appears in numerous partial equilibrium analyses, and in the general equilibrium work of Radner (1972, 1980) and Sondermann (1974) among others. It is justified by the fact that firms have to decide on many issues, for which stock prices are uninformative and consultation of shareholders impossible, but where consistent decisions remain a goal.

[1] The inequality (3) follows from substituting $\hat{\mathbf{y}}^j$ for $\mathbf{y}^j$ in (1)–(2), keeping all other right-hand variables unchanged.

The approach of this third group has been criticised on the already mentioned grounds that firms have no physical identity in which risk preferences could be rooted. Also, provision should be made for the utility function of a firm to reflect its ownership structure, the resulting shareholders' preferences and/or the information contained in stock prices, when appropriate; that is, provision should be made for integrating the three approaches.

Let me quote the conclusions of my 1982 survey:

'A firm contemplating to start a new line of products may, or again may not, find in share prices on the stock market a clearcut evaluation of the new venture. If the products are genuinely new, or if information about the production plans of existing firms is fragmentary, the share prices will not provide the desired evaluation. These prices may still contain useful partial information, for instance about the advisability of buying up existing production facilities instead of building a new plant from scratch. If the decision to start the new line has major consequences for the firm's future, it may be appropriate to consult shareholders, for instance at an annual meeting; or to consult 'the stock market', for instance by incorporating the new venture separately, and undertaking the investment only if the issue of new shares is fully subscribed[1]. Once a decision to start the new line of products has been reached, many ancillary decisions remain to be taken, concerning production technology, product design, advertising, and so on. These ancillary decisions will be vested with management, and are not apt to be guided by share prices or shareholders' preferences. These decisions involve all sorts of uncertainties. Experience suggests that managers of many firms display a degree of risk aversion in reaching such decisions; otherwise, we could not account for the existence of insurance policies designed specifically to cover business risks, like machine breakdowns, ship wreckages, bank hold-ups, and so on[2].

On these grounds, I feel that theories based on utility functions for business firms do have an operational justification, at least as a first approximation, *if they make room for the information conveyed by share prices and an occasional consultation of shareholders.*' (Drèze, 1982, p. 44)

Leland (1978) has suggested that the manager of a firm could be himself a shareholder. If the utility of the firm is simply the utility of the manager, then maximisation of that utility implies Pareto optimality for the shareholders, since the manager is one of them. And we have seen that Pareto optimality for the shareholders entails consistency with market value maximisation, to the extent that it is defined[3]. In this way the three approaches become 'nested', and

[1] This form of consultation of potential shareholders is limited in scope, however, because the announcement that the venture will be discarded if the issue is undersubscribed distorts the information content of shareholders' responses.

[2] The possibility of risk neutrality through the spreading of given risks among a large number of shareholders was mentioned at the end of Sec. 1.5 (Drèze 1982). It should however be noted that liquidity constraints may lead a risk neutral firm to behave as if it were risk averse, with a risk aversion factor determined by technology; see Böhm (1980, Sec. 3.1.2) or Drèze and Marchand (1976, Sec. 5).

[3] This consistency will also obtain if the firm owns a portfolio of shares, as in the model of Sondermann (1974).

the preferences of a single individual – the manager – define a utility function for the firm, which meets the requirements of the three approaches just reviewed.

There remain problems, however. First, the identity of the manager is exogenously given, independently of the ownership structure of the firm. Thus a majority of shareholders could not appoint its own manager. Second, it is assumed that the manager is a shareholder of the firm, without explicit reference to his portfolio choices. In particular, diversification between the risks on labour income and on property income might discourage the manager from owning stock in his own firm. Third, although it is true that a manager shareholder maximising his own utility brings about Nash-Pareto optimality, it may still be the case that his preferences are quite at variance with those of a majority of shareholders. For instance, he could be a very competent manager, while shunning risks in so far as his own finances are concerned. It would then be natural for him to consult shareholders on decisions involving major uncertainties, while holding no stock and working for a fixed salary. Should he fail to do so, it would be natural for the shareholders to impose a policy line, or to replace him.

These shortcomings point to the desirability of seeking to integrate the three approaches in a way which brings more explicitly to the foreground the interaction between the portfolio choices of consumers, the decision criteria of the firms and the practice of management.

## III

Recently, I have followed another avenue towards integrating the three approaches to decision criteria for the firms. It is in a sense more institutional, starting as it does from a naïve observation of some legal and administrative aspects of business management. This alternative avenue was developed in the 1983 Jahnsson Lectures, to be published by Basil Blackwell under the title *Labour Management and Labour Contracts* (Drèze, 1985*b*). It may be summarised as follows.

Modern corporations are typically organised under legal systems which vest the ultimate authority with shareholders, reaching majority decisions at general meetings (with the possibility of qualified majorities for special decisions, like modifications to the company's statute). Because general meetings of shareholders are unwieldy, provision is made for partial delegation of authority to a Board of Directors, comprising a small number of members, elected by the shareholders. The Board typically decides about its own *modus operandi*. Each year, the Board requests from the general meeting of shareholders endorsement of its decisions. Because Directors could not conveniently meet every day, the Board in turn delegates some authority to managers, under the responsibility of one or more members of the Board (the President, or Deputy Directors,...).

It is somewhat surprising, but definitely intriguing, that this mode of organisation can easily be fitted into the abstract formalism of general equilibrium theory (under an assumption which does not seem unduly restrictive), and covers earlier approaches as special cases.

To start with a simple example, suppose that a corporation has been formed, with the provision that the Board of Directors will consist of the 4 largest shareholders, and that decisions (of some significance) must be approved by all the directors and by a majority of shareholders. An abstract version of this example runs as follows.

Given the ownership of firm $j$, as defined by the $I$-tuple of ownership fractions $\boldsymbol{\theta}^j = (\theta^{1j}, \ldots, \theta^{ij}, \ldots, \theta^{Ij})$ resulting from the portfolio choices of the consumers, there exists for firm $j$ a *Control Group* (the Board of Directors), say $I^j(\boldsymbol{\theta}^j)$, a subset of the shareholders (a subset of $\{1, \ldots, i, \ldots, I\}$, with $\theta^{ij} > 0$ for each $i$ in the subset). For two feasible production plans of firm $j$, say $\mathbf{y}^j$ and $\hat{\mathbf{y}}^j$ in $Y^j$, we shall say that $\mathbf{y}^j$ is *preferred to* $\hat{\mathbf{y}}^j$ *by firm j if* $\mathbf{y}^j$ *is preferred to* $\hat{\mathbf{y}}^j$ *by all the members of the control group and by some set* $\hat{I}^j(\boldsymbol{\theta}^J)$ *of shareholders representing more than fifty percent of the ownership*[1]; more concisely, $\mathbf{y}^j$ is preferred to $\hat{\mathbf{y}}^j$ by firm $j$ (given $\boldsymbol{\theta}^j$) if it is preferred by *some majority group* (of shareholders) *including* $I^j(\boldsymbol{\theta}^j)$. (An important aspect of the definition is that the majority deciding to choose $\mathbf{y}^j$ over $\hat{\mathbf{y}}^j$ may be different from the majority choosing $\mathbf{y}^j$ over some third alternative $\tilde{\mathbf{y}}^j$ – but all members of the same control group must in either case belong to the decisive majority.)

This stipulation is sufficient to give content to condition (*iii*) in the definition of an equilibrium of the stock-market economy, namely:

(*iii′*) $\mathbf{y}^j$ is a best production plan in $Y^j$, i.e. there does not exist an alternative plan $\hat{\mathbf{y}}^j$ and a majority group of shareholders $\hat{I}^j(\boldsymbol{\theta}^j)$ including the control group $I^j(\boldsymbol{\theta}^j)$, such that $\hat{\mathbf{y}}^j$ is preferred to $\mathbf{y}^j$ by all members of $\hat{I}^j(\boldsymbol{\theta}^j)$[2].

I now submit that the precise rules determining who belongs to the control group (who becomes a Director), *as a function of the ownership of the firm*, may properly be treated as an exogenous primitive datum by the general equilibrium theorist. His problem is only to verify whether or not the rules are consistent, meaning here conducive to existence of an equilibrium, under reasonable assumptions about the other primitive data of the economy.

To elucidate the existence issue, let me begin with a simple intermediate proposition. Ignoring momentarily the dependence of the control group on the ownership of the firm, I shall consider first the case where for each firm the control group is given *a priori* (as was the case for the manager, in the work of Leland reviewed in Section II above), and members of the control group always own a positive share of the firm.

*PROPOSITION: Assume that for each j, the control group is given a priori; then, under standard* (TV) *assumptions, there exists an equilibrium for the stock-market economy.*

This proposition, proved in the manuscript of the 1983 Jahnsson Lectures, is a new existence result, not covered by those of Drèze (1974 *b*) or Grossman and

[1] The simple majority rule ($\sum_{i \in \hat{I}^j(\theta^j)} \theta^{ij} > 0.5$) could be replaced by any other majority rule.

[2] In applying this definition, it is understood as before that shareholder $i$ prefers $\hat{\mathbf{y}}^j$ to $\mathbf{y}^j$ if and only if he prefers the resulting consumption plan, i.e. if and only if $u^i(\mathbf{x}^i + \theta^{ij}\hat{\mathbf{y}}^j - \theta^{ij}\mathbf{y}^j) > u^i(\mathbf{x}^i)$.

Hart (1979) which ignore majority approval but require transfers among shareholders of a kind not frequently encountered (or imaginable, for that matter) in the financial world.

It is perhaps not a surprising result, given that decisions within each firm are reached by majority voting with a fixed set of veto players. It is well known that existence of veto players eliminates the Condorcet paradox of voting, a paradox stressed in our context by Gevers (1974). Majority voting with veto leads to a decision criterion for each firm which is a partial ordering, free of intransitivities (though not necessarily transitive), and consistent (where defined) with the preferences of veto players. These properties, combined with standard TV assumptions, are sufficient to prove existence. The technical argument follows the approach of Shafer and Sonnenschein (1975) to 'Equilibrium in Abstract Economies without Ordered Preferences'.

Of course, it is not congruent with general equilibrium methodology to assume that the control group is given *a priori*, independently of the ownership of the firm. As repeatedly stressed before, we would like the control group to reflect ownership (as in my example of a Board of Directors comprising the 4 leading shareholders). To handle this more general case, two assumptions are needed – a minor technical one, and a major institutional one.

The technical assumption (hereafter, Assumption *B*) is meant to preclude indeterminacies arising if a firm becomes inactive, and nobody cares to hold its stock. We still need to verify that inactivity is a 'best' plan for that firm. The easiest way of covering that loophole consists in assuming that *some* initial shareholders of the firm never divest themselves completely of their stock. A more satisfactory (weaker) assumption, which is also more demanding technically, only requires that somebody always be prepared to hold the stock *at a zero price.*

The important assumption is the institutional one, which imposes a degree of continuity in the exogenous rule determining how changes in ownership affect the composition of the control group. Here are three equivalent statements of the proposed assumption, couched in increasingly abstract terms[1].

*ASSUMPTION A.1: Let* $I^j(\theta^j)$ *be the control group associated with the ownership vector* $\theta^j$; *for all* $\hat{\theta}^j$ *sufficiently close to* $\theta^j$, $I^j(\hat{\theta}^j)$ *is included in* $I^j(\theta^j)$.

*ASSUMPTION A.2: For all i, the set of ownership vectors* $\theta^j$ *such that consumer i belongs to* $I^j(\theta^j)$ *is closed.*

*ASSUMPTION A.3: The correspondence* $I^j$ *which associates with every ownership vector* $\theta^j$ *a control group* $I^j(\theta^j)$ *is upper semi-continuous in the discrete topology.*

These three statements are formally equivalent and will be referred to as Assumption *A*. A general existence result is:

*THEOREM: Under standard* (TV) *assumptions and assumptions A, B, there exists an equilibrium for the stock-market economy.*

[1] I am grateful to Bernard Cornet for suggesting the more abstract formulations.

In this proposition, control groups reflect ownership, and the decisions of the firms reflect ownership and control. This is a genuine 'general equilibrium' result. It rests on reasonable assumptions, and covers most previous results as special cases (as the reader can easily verify)[1].

Before stating further generalisations, a world of comment about assumption $A$ is in order. Perhaps the more transparent statement is $A$ 2, which says: if you belong to the Board of Directors for every ownership vector in a sequence converging to $\theta^j$, then you also belong for $\theta^j$ itself. For example, if you belong to the Board provided you own *more than* 5% of the shares, you should also belong if you own *exactly* 5%. This requirement is not vacuous – but it does not seem unduly restrictive either; and it is interesting to discover that it is consistent with decentralised portfolio choices, and decentralised decisions by firms, to the extent of implying existence of equilibria.

Examples of statutes verifying Assumption $A$ include Boards of Directors consisting of every shareholder holding at least $\alpha$ % of the stock; or consisting of the $k$ leading shareholders; or consisting of the set of leading shareholders holding together $\beta$% of the stock, and so on (with appropriate tie-breaking rules). These examples are illustrated in Figs. 1, 2 and 3 respectively, for the case of three shareholders – see the Appendix.

The main drawback of the proposed decision criterion (majority vote among shareholders with veto right for Directors) is the requirement of *unanimity among the Directors.* This is of course a much weaker requirement than unanimity *among the shareholders.* Also, over the small group of Directors, side payments could (and sometimes do) take place to facilitate agreement. More importantly, perhaps, the proposed criterion is easily generalised to a two-tier majority rule. For instance, the Board of Directors could reach decisions through majority voting, with a veto right for the Chairman of the Board, or for some particular subset of the Board. This extension is immediate, when the subset with veto power is again determined in a manner verifying Assumption $A$. (Note that a veto right is conveniently exercised by controlling the agenda of meetings and putting to a vote preferred changes only.)

An interesting extension, which available time does not permit discussing in details, consists in recognising formally the existence of *partial delegation of authority.* Thus, the shareholders might endow the Board of Directors with *final authority over a set of issues,* whereas issues outside that set would still require approval by a majority of shareholders. In turn, the Board could delegate to management final authority over a subset of issues from its own sphere of authority.

Work in progress shows that such delegations are consistent with existence of decentralised equilibria, provided two conditions are met:

(*i*) Every person or group receiving delegation belongs to the group granting delegation (i.e. the manager is a Director and Directors are shareholders);

(*ii*) the set of issues over which delegation is granted, relative to a given

[1] In particular, if $I^j(\theta^j)$ consists of all consumers owning at least 'one share' of stock, we are back to the Nash-Pareto criterion of the second group; if $I^j(\theta^j)$ consists of a single manager shareholder (possibly identified *a priori*), we are back to Leland's version of a utility function of the firm.

production plan, is a convex subset of the production set (actually, of the set for which the body granting delegation has final authority), and depends *continuously* upon the reference plan.

Under all the forms of *control* and *delegation* referred to here, the decision criterion of a firm defines a partial ordering over production plans. In general, the ordering is not complete nor representable by a concave utility function. But it satisfies the two conditions used in the existence theorem for abstract economies; namely, a continuity condition ('the preference correspondence has open graph') and a convexity condition ('a production plan never belongs to the convex hull of the set of plans preferred to it'). These properties are sufficient for many purposes, a claim illustrated by the following application.

## IV

As noted above, the model of Diamond is highly streamlined. Although it captures an important aspect of private ownership economies, namely the interaction of production decisions and portfolio choices, still it ignores essential aspects of business organisations. In particular, it leaves out labour altogether, to concentrate exclusively on capital. It is striking that, in the sixties and early seventies, so much attention was devoted to portfolio problems, and so little attention to labour contracts, in spite of the fact that uncertainties about labour income are much more significant than capital gains or losses, for most people[1].

This is perhaps revealing of the geographical concentration of our GET-set. In my experience, when European economists from different countries meet socially, there comes a time when they discuss salaries. When American economists meet socially, they eventually discuss the stock market; whereas Israëli or Indian economists discuss credit conditions. (Incidentally, this observation relates to Milton Friedman's (1964) celebrated 'Tenured Income Hypothesis', claiming that the standard of living of University Teachers of Economics is the same all over the world, but the number of jobs they hold is inversely proportional to the country's wealth. Friedman may well be right after all, in spite of the facts that living conditions are more attractive in Bath than in Buffalo and that Jerusalem is a safer city than Chicago.)

It was left for income-conscious European economists, like Costas Azariadis (1975) and Martin Baily (1975), working in the efficient environments of North-American Universities, to restore priorities by drawing attention to labour contracts as instruments of risk sharing.

I need not remind you of the contents of implicit contracts theories of 'wages and employment under uncertain demand'. Oliver Hart (1983) devoted his Review of Economic Studies Lecture at your 1983 meeting to this topic. Let me however remind you of one significant assumption in the presentation by Oliver Hart and in all the antecedent literature: each firm has well-defined preferences over state distributions of profits, representable by a concave utility function ....

[1] On this point, see Drèze (1979).

It is an obvious task for general equilibrium theorists to bring the partial equilibrium analysis of labour contracts in harmony with production decisions of the firms and portfolio choices of their owners; in other words, to introduce labour inputs in the Diamond model, and study the existence and properties of equilibria defined simultaneously for the stock market, the markets for labour contracts, and the decentralised decisions of firms and households; with the firms now choosing labour contracts as well as production plans, and with the households now supplying labour as well as holding portfolios. This is clearly a more ambitious, yet realistic programme. It belongs to the core of my 1983 Jahnsson Lectures. It is germane to the 'institutional' approach developed above for decision criteria of the firms, since negociated labour contracts are the most commonplace institutional device for private risk-sharing arrangements between labour and capital in Western economies.

There are by now a number of alternative models of labour contracts in the literature.[1] Some are more easily fitted than others into the general equilibrium framework considered here. A simple model goes as follows.

There is a single type of labour.[2] Each firm offers a labour contract, which specifies the quantity of work to be performed in the initial period, as well as in the future period under every state (that is, it specifies a vector of state-dependent labour times or retention probabilities); the contract also specifies the hourly wage, both now and tomorrow in every state. The decision of the firm becomes a pair consisting of a production plan and a labour contract. Such a decision implies an initial investment or dividend, as the case may be; and a vector of state-dependent future dividends, defined for each state by the value of output minus the wage bill. A decision $d^j$ is best for firm $j$ if there exists no alternative decision $\hat{d}^j$ deemed preferable by a majority of shareholders including the control group, *and by a majority of workers as well.* This approach looks at labour contracts as negotiated with labour, and subject to approval by majority voting at a meeting of all workers. (If one wishes, a control group with veto rights can be introduced among the workers as well, for instance a group consisting of the union representatives. Other approaches to labour's endorsement of the contract are conceivable. For instance, a new contract could be chosen in the convex hull of all existing contracts, thereby generalising the standard condition of a satisfactory utility level for identical workers; or the contract could have to be such as to generate an adequate labour supply. See Drèze (1985*b*) for a precise statement.)

At the same time, households make joint decisions about their portfolios of assets and their labour supply, choosing among the labour contracts offered by the different firms. Pending wage rigidities, easily handled along the now familiar lines of equilibrium theory with price rigidities and quantity rationing[3], *an equilibrium for the stock-market economy with labour contracts* consists

[1] See Azariadis (1979) or Ito (1982) for recent surveys.

[2] Explicit treatment of several types of labour raises no difficulty. Also, workers are *not* assumed identical in what follows.

[3] Cf. Drazen (1980) or Grandmont (1977).

of a vector of share prices **p**, and a set of decisions for each consumer and each firm, such that:

(*i*) The stock market clears;

(*ii*) the market for labour contracts *in each firm* clears;

(*iii*) each consumer is in equilibrium (in terms of portfolio choice and labour supply), given the share prices **p**, the dividend prospects of each firm, and the terms of the available labour contracts;

(*iv*) each firm is in equilibrium, in the sense that: no alternative feasible decision (production plan and labour contract) would be preferred by a majority of shareholders and a majority of workers, including the (endogenously defined) control group(s).

*Under assumptions similar to those of the previous theorem, such an equilibrium exists!*

On several previous occasions (Drèze, 1976, 1979), I have characterised firms as fulfilling three roles: they realise physical investments, that match consumer savings; they create jobs, which consumers fill; they produce commodities, which consumers buy. The work reviewed here integrates the first two aspects in a general equilibrium theory. Formal analysis of the third aspect, in a general equilibrium model with uncertainty and incomplete markets, remains an open challenge.

## V

What is to be concluded from this presentation? I will offer six brief remarks, on the following theme:

If we are willing to assimilate general equilibrium theory to a cardboard box rather than to an envelope, firms can fit into such a box without being flattened out – and general equilibrium theory is not an empty box! Let me elaborate in steps.

(1) The existence results quoted above indicate that firms can be fitted into general equilibrium theory, in a way which draws a reasonable boundary between economic analysis and the exogenous environment, and which leads to verify the overall consistency of an economic analysis based on relatively weak assumptions.

The boundary has been shifted by treating as exogenous the rules whereby control of a firm is related to its endogenously determined ownership, instead of treating control as altogether exogenous.

(2) Of course, the results presented here should not be viewed as the final word on the subject. My only claim is that these results cover previous approaches as special cases. This justifies taking stock of these results, in the hope that they may stimulate further research in an area which should, in my opinion, retain due attention.

(3) The resulting decision criteria of the firm are somewhat hybrid, reflecting as they do nested majority decisions with veto rights, possibly combined with partial delegation of authority (and tie-breaking rules). In general, the resulting preferences will not be representable by a concave utility, and will depend upon

the overall allocation (portfolio choices and production plans of the other firms). But the preference correspondences have open graph, and a decision never belongs to the convex hull of the set of preferred decisions.

The implications of this kind of preferences for other problems (in particular, properties of supply correspondences or of efficient labour contracts) *remain to be investigated.*

As an immediate implication, the set of equilibria for the stock-market economy will be larger and more diversified than, say, under spanning – the set may be more like a box with a non-empty interior than like an envelope with measure zero.

(4) The equilibria under consideration are Nash-Pareto optima, but not full Pareto optima, as was to be expected given the non-convexity of the problem, already stressed ten years ago.

(5) The equilibria under consideration have a competitive flavour, in that portfolio choices are based on stock prices and announced production plans. Although the rules determining membership in the control groups (Boards of Directors) are typically public knowledge, no allowance is made for portfolio choices *aimed at participation in control* – a question worth pursuing. Similarly, delegation of authority should be linked to the growing literature on 'principal-agent' problems.[1]

(6) The approach followed here, and in particular the technical treatment of the existence problem in terms of abstract economies without ordered preferences, is quite robust. This has been illustrated by the ease with which labour contracts could be fitted into the Diamond model – a step of some substantive interest in itself. *There remain many open issues* in temporary general equilibrium theory calling for analysis with uncertainty and business decisions. Hopefully, a robust approach to these decisions may permit new developments.

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## Appendix

The figures below illustrate rules, for defining control groups, which satisfy Assumption *A*. The figures are drawn for 3 shareholders, whose ownership fractions $(\theta^1, \theta^2, \theta^3)$ in some firm add up to 1 and thus belong to the triangle with vertices at $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$; i.e. to the unit simplex of $R^3$.

In Fig. 1, the rule defining the control group is: 'shareholder $i$ belongs to the control group $I(\theta)$ if and only if $\theta^i \geqslant 0.25$'.

In the inner triangle with heavy boundary, this condition is satisfied for each $i = 1, 2, 3$. That triangle (boundary included) is thus the set of ownership structures $\theta$ for which $I(\theta) = \{1,2,3\}$. Immediately to the right of the inner triangle, one finds a trapeze defining the set of ownership structures for which $I(\theta) = \{1,2\}$. That set is closed (contains its boundary) on three sides, but it does

[1] Cf. Shavell (1979).

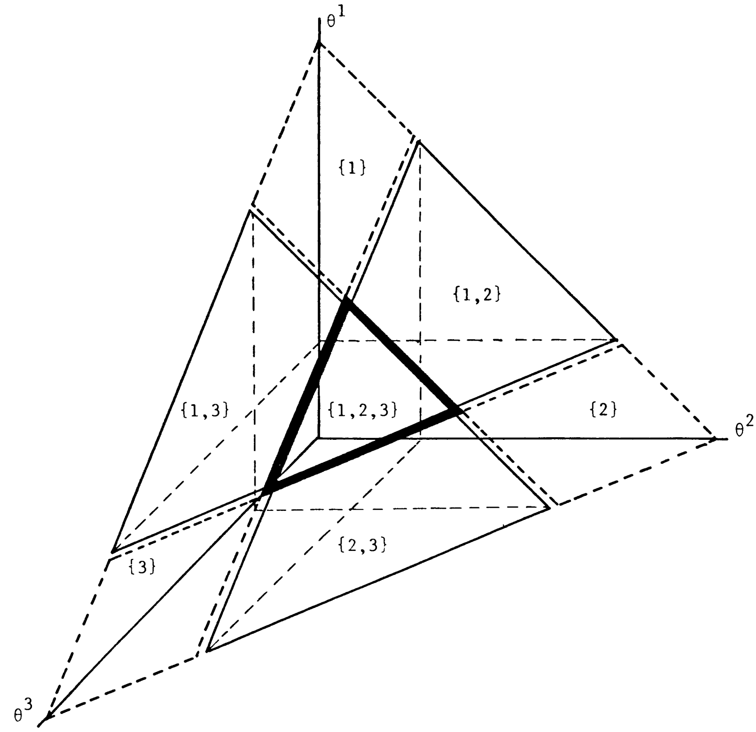


Fig. 1

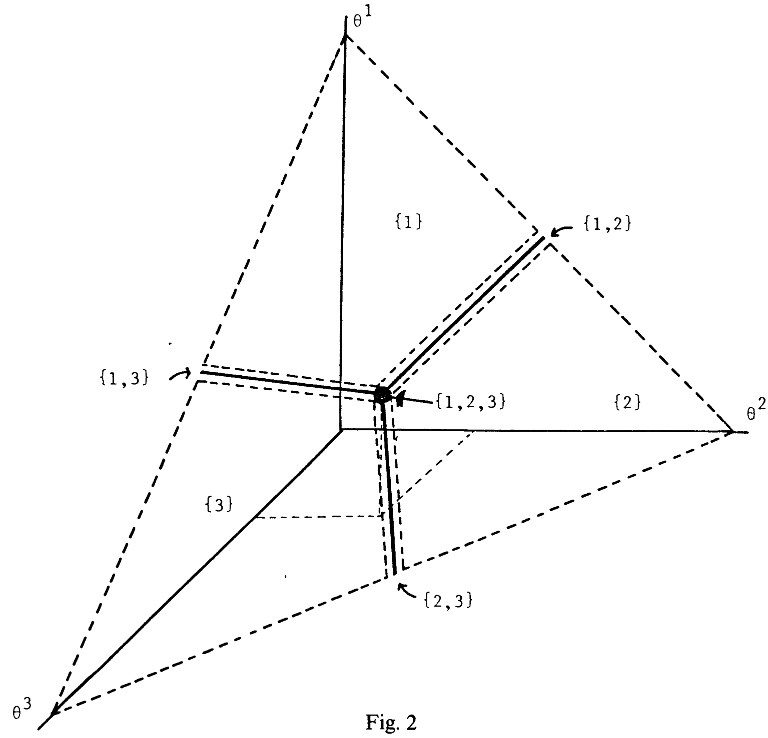


Fig. 2

not contain the face which belongs to the boundary of the inner triangle. (On that face, $\theta^3 = 0.25$ so that 3 belongs to $I(\boldsymbol{\theta})$.) Similar remarks apply to the trapezes for $\{1,3\}$ and $\{2,3\}$. Finally, the lozenges containing the vertices of the simplex correspond to one-person control groups. They are closed on the two sides which also belong to the boundary of the simplex, but open on the interior sides.

In Fig. 2, the rule defining the control group is: '$I(\boldsymbol{\theta})$ consists of the single largest shareholder; if two or more shareholders have equal holdings, unsurpassed by others, they all belong'. Formally, $I(\boldsymbol{\theta}) = \{i \mid \theta^i \geqslant \theta^j \forall j\}$.

The construction is analogous to that of Fig. 1. $I(\boldsymbol{\theta}) = \{1, 2, 3\}$ at the single point (1/3, 1/3, 1/3), marked with a heavy dot. $I(\boldsymbol{\theta}) = \{1,2\}$ along the solid line from (1/3, 1/3, 1/3) to (1/2, 1/2, 0). Outside of the set of measure zero corresponding to ties, there is a single person in the control group. Thus, 1 belongs to that group for all values of $\theta$ in the closed tetrahedron corresponding to the upper part of the figure; he is the only member of the control group for all values of $\theta$ in the interior of that tetrahedron or on the sides contained in the boundary of the simplex.

In Fig. 3, the rules is: '$I(\boldsymbol{\theta})$ is the smallest group of shareholders owning together at least 1/3 of the shares; if two or more distinct groups meet this condition, $I(\boldsymbol{\theta})$ is the union of their members'. The interpretation of the figure is left to the reader.

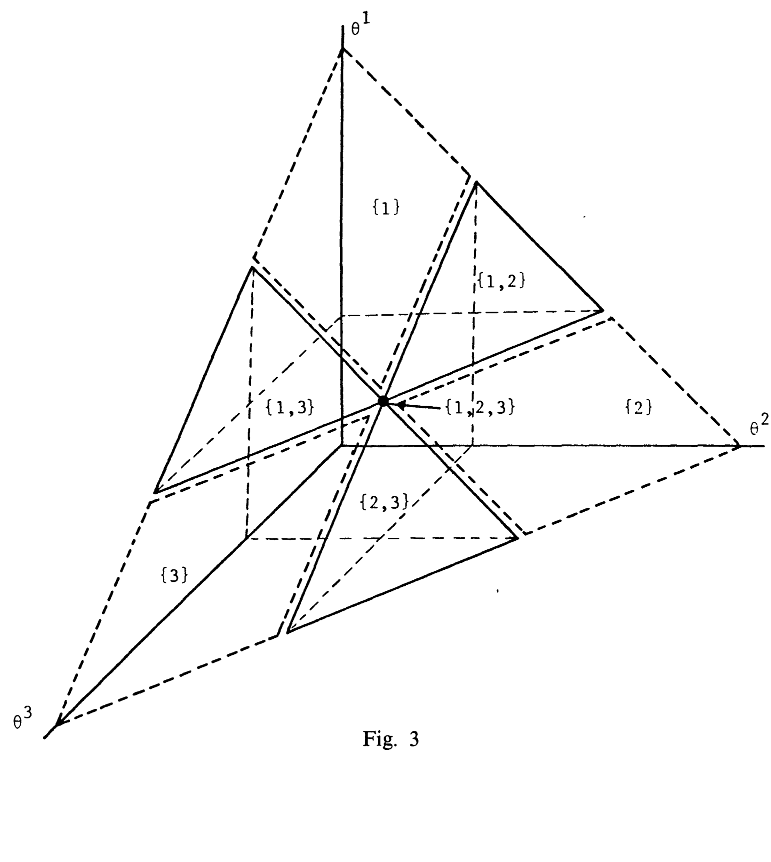


Fig. 3

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